ANALYSIS OF ELECTROMAGNETIC FORCES ON MAGNETICALLY SUSPENDED HIGH-SPEED TRAINS[1]

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Abstract. High-speed superexpresses (HSST) developed by Japanese airlines (JAL) are based on the electrodynamic principle of magnetic suspension. The track contains short-circuited coils and interaction between them and superconductive coils in the vehicle produces its suspension. The paper includes a mathematical model for traction electrodynamic suspension device HSST represented by a system of linear differential equations with coefficients varying in time. Numerical analysis of this model yields the velocity-dependent lift and drag forces acting on the system. The time distribution of the lift force exhibits certain oscillations that may be suppressed by suitable placement of several superconductive levitation windings in the vehicle. The results obtained are in a good agreement with the knowledge found by various authors on prototype vehicles.

Key words. Maglev, high speed ground transportation, electrodynamic suspension, lift force in maglev, drag force in maglev.

1. INTRODUCTION

Problems associated with magnetic levitation (known as *maglev*) were intensively examined in the last forty years (particulars see, for example, in [1], [2]). Its most important applications can be found in transport. Developed were magnetic superexpresses HSGT (*High Speed Ground Transportation*), i.e. magnetically levitated high-speed trains that move above the track and their velocity can exceed even 500 km/h. Several levitation transport systems have been designed; technically were realized, however, only two of them: electromagnetic system and electrodynamic system. The first one is projected by German companies and is known under name *Transrapid*. Particularly Japanese companies developed the second one called MLU (Magnetic Levitation Unit). References dealing with magnetic superexpresses are obviously influenced by patent protection; they usually have popularizing or advertising character or deal only with some technical details of suspension systems. Synthetic works on theory and projection of suspension systems in transport have not been published so far.

The authors present theoretical investigation of forces acting in an electrodynamic suspension system. They started from the *static model* of electrodynamically levitated system that works with straight movement of the vehicle with a constant velocity. Determination of forces is necessary for building a *dynamic model* of magnetic suspension that represents the starting point for optimization of the system and, consequently, for its design.

Most papers aimed at the electrodynamic suspension systems present their force characteristics that were found, however, only experimentally (see, for example, [3]). Only few papers deal with their theoretical investigation. In the next text we will use the circuit conception.

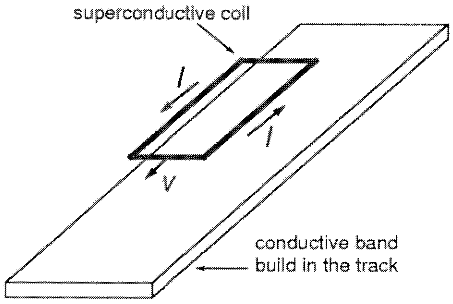

Fig. 1. Moving superconductive magnet above conductive band.

2. ELECTRODYNAMIC SYSTEM OF MAGNETIC SUSPENSION

Physical fundamentals of the electrodynamic system of magnetic suspension (EDS - Electrodynamic Suspension) follow from Fig. 1. An electrically conductive band and a DC current carrying coil that moves above the band at a velocity v represent the main parts. Magnetic field produced by the moving coil generates eddy currents in the conductive band and their magnetic field is in interaction with the primary field of the coil. Thus, the coil is affected by a force. In order that the lift force is sufficiently high, magnetic field of the coil must reach extreme values. This can be achieved only at extreme

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high currents in the coil and this is possible only when the coil is superconductive.

A more detailed description of various arrangements of coils in EDS systems, details about solution of the LSM drive and further technical data concerning electrodynamically suspended superexpresses are not the subject of this paper and readers may find them in [1], [3], [4] and [5].

3. ANALYSIS OF FORCES IN THE LEVITATION SYSTEM – A CONTINUOUS MODEL

The levitation system consists of coils arranged by Fig. 2. The track contains built-in a series of the same rectangular coils, each of them being formed by one short-circuited turn. We will call them *unmovable coils* and denote by indices 1, 2, … . In the vehicle above the track we suppose one rectangular superconductive coil that moves at a velocity $v = v_x =$ const that carries direct current I of high value. This coil is called *movable* coil and will be denoted by index 0. The movable coil produces strong magnetic field in its neighborhood characterized by time variable flux linkages with particular unmovable coils in the track. Magnetic flux Φ_k that is linked with the k-th unmovable coil can generally be expressed as

$$\Phi_k(t) = M_{k0}(t)\, I\,, \tag{1}$$

where the mutual inductance between the movable and k-th unmovable coil is

$$M_{k0}(t) = M_{10}(vt - \lambda_k)\,, \tag{2}$$

function M_{10} being the mutual inductance between the first unmovable coil and movable coil 0. We can determine it from the Neumann formula

$$M_{10} = \frac{\mu_0}{4\pi} \oint_{c_1} \oint_{c_2} \frac{\mathrm{d}\mathbf{l}_1 \mathrm{d}\mathbf{l}_2}{\mathbf{r}_{10}}\,, \tag{3}$$

where: - curves c_1 and c_0 are rectangles formed by both coils 1 and 0,

- $\mathrm{d}l_1$ and $\mathrm{d}l_0$ are oriented length elements of these curves and
- r_{10} is their distance.

Quantity λ_k in (2) is given as

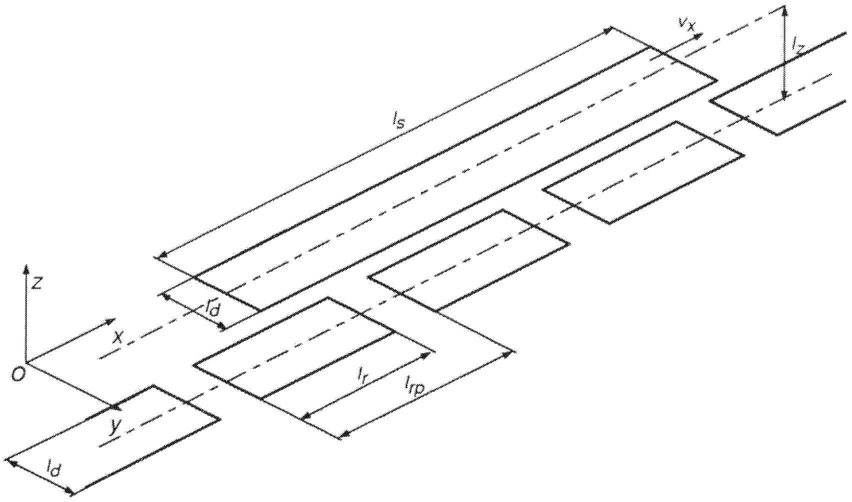

Fig. 2. Arrangement of the movable coil (placed in the vehicle) and a system of unmovable coils (in the track).

$$\lambda_k = (k - 1)\, l_{\mathrm{rp}}\,, \tag{4}$$

where k denotes the order in the row of the unmovable coils and l_{rp} is the distance between the unmovable coils, see Fig. 2. Currents in the unmovable coils are described by a system of ordinary differential equations in the form

$$R_k i_k + L_k \frac{\mathrm{d}i_k}{\mathrm{d}t} - M_0 \left(\frac{\mathrm{d}i_{k-1}}{\mathrm{d}t} + \frac{\mathrm{d}i_{k+1}}{\mathrm{d}t} \right) + \frac{\mathrm{d}\Phi_k}{\mathrm{d}t} = 0 \tag{5}$$

where R_k and L_k are the resistance and inductance of the k-th unmovable coil and M_0 is the mutual inductance between the neighboring unmovable coils (k = 1, 2, 3, …). It can be again derived from the Neumann formula (3). The number of neighboring coils that will be taken into account depends on the chosen accuracy. Solution to differential equations (5) will provide currents in all unmovable coils 1, 2, 3, ….

The force acting between the unmovable coils and movable coil has two components: lift force $F_z(t)$ and drag force $F_x(t)$. Both components can be derived from the magnetic field energy. As known, they depend only on the derivative of the mutual inductance in their directions. Thus, the lift force is

$$F_z(t) = I \sum_k i_k \frac{\partial}{\partial z} M_{k0}(x, z, t) \tag{6}$$

and drag force

$$F_x(t) = I \sum_k i_k \frac{\partial}{\partial x} M_{k0}(x, z, t)\,, \tag{7}$$

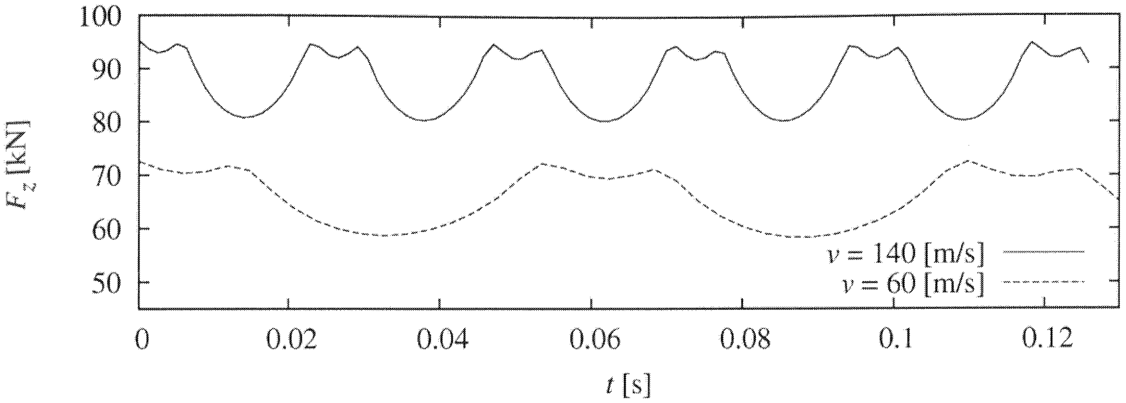

Fig. 3. Time evolution of the instantaneous values of the lift force at velocities $v_x = 60$ m/s and $v_x = 140$ m/s.

where M_{k0} is given by (2). The time evolution of both components is periodical with period T, with a significant constant part that can be calculated as an average value of both forces

$$\overline{F_z} = \frac{1}{T}\int_{t_0}^{T+t_0} F_z(t)\,\mathrm{d}t\,, \tag{8}$$

$$\overline{F_x} = \frac{1}{T}\int_{t_0}^{T+t_0} F_x(t)\,\mathrm{d}t\,. \tag{9}$$

The period T depends on velocity v_x of the movable coil and distance l_{rp} between particular unmovable coils. It can be expressed as

$$T = \frac{v}{l_{rp}}. \tag{10}$$

4. ILLUSTRATIVE EXAMPLE, CHARACTERISTICS

The numerical solution has been performed for the following input data (with respect to [6]):

- dimensions of the movable and unmovable coils (Fig. 2):

l_s = 9.00 m, l_d = 0.50 m, l_r = 3.25 m, l_{rp} = 3.33 m, l_z = 0.25 m,

- current in the movable coil: I = 200 kA,
- parameters of the movable coil: resistance $R = 25.4\cdot 10^{-6}$ Ω, self-inductance $L = 3.93\cdot 10^{-6}$ H and mutual inductance between two neighboring unmovable coil determined from (3): $M_{12} = 0.098\cdot 10^{-6}$ H.

Integration of the system of differential equations (5) then provided time evolution of currents in the unmovable coils. Equations (6) and (7) were used for computation of the lift and drag forces for various velocities v_x of the movable coil. Their graphs for v_x = 60 m/s and v_x = 140 m/s are given in Fig. 3. The average value of the lift force $\overline{F_z}$ determined from (8) is in Fig. 4 and the average value of the drag force $\overline{F_x}$ determined from (9) is in Fig. 5.

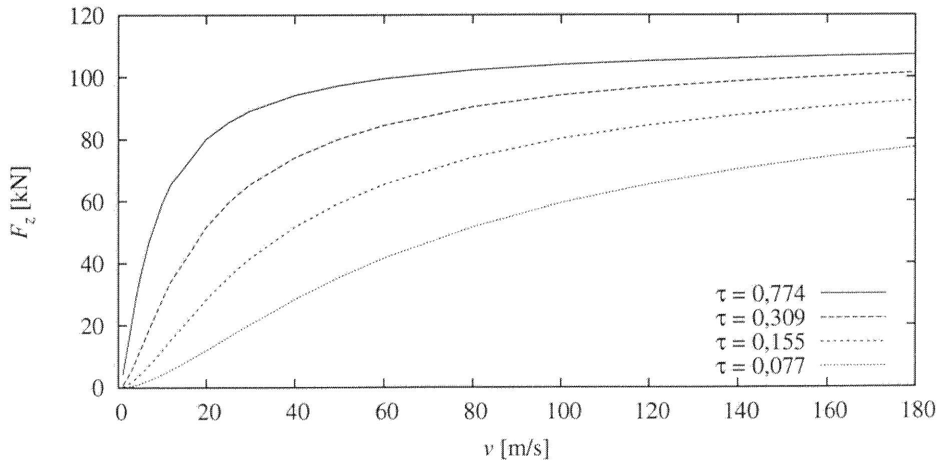

Fig. 4. Average value of the lift force as a function of velocity, for various time constants $\tau = R_k/L_k$ of the unmovable coils.

5. CONCLUSION

The *static model* of suspension of an electrodynamic system became the first step for formulation of a computational algorithm for determining the force characteristics. The numerical computation leads to results that are in a very good qualitative agreement with the knowledge presented in relevant references. The knowledge can be used not only at the design of unmovable coils built in the track and proposal of the drive (an knowledge presented in relevant references. The knowledge can be used not only at the design of unmovable coils built in the track and proposal of the drive (an LSM with a long stator), but particularly at construction of the whole dynamic model that will be used for solution of overall dynamics of the whole system with electrodynamic suspension. The *dynamic model*, however, includes an influence of other factors such as side forces (produced by curvature of the track or by wind) and makes it possible to solve all transients or eventual vibrations of the system.

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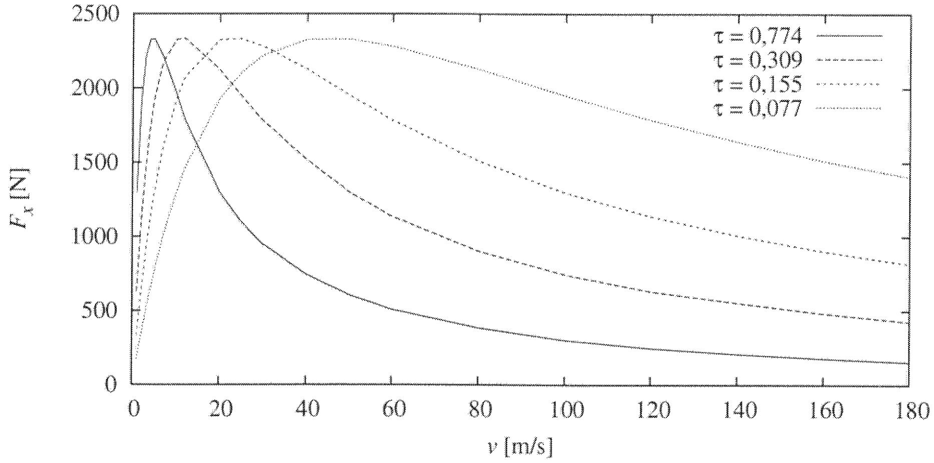

Fig. 5. Average value of the drag force as a function of velocity, for various time constants $\tau = R_k/L_k$ of the unmovable coils.